Please answer all 3 questions and subquestions below.

Problem 1

(a). In Kyle's dealer model (lecture 5) we obtained a price function $p = \mu + \lambda q$, which implies that the spread goes to zero as q goes to zero. In Glosten's model of the limit order book (lecture 7), this is not the case: the spread is strictly positive even for very small order sizes. Explain intuitively this difference.

Suggested solution. As talked about in class, in Kyle's model the total order size is known, and therefore also the extent of adverse selection. When the order size goes to zero, so does adverse selection. In the limit order book on the other hand, the first limit order may execute against a very large market order (indicating a great deal of adverse selection), but this is unknown at the time the order is posted. Thus, even the first unit of the first limit order in the book is subject to substantial adverse selection.

(b). The textbook notes on page 263 that fragmentation causes the limit order book to be deeper at each tick. Discuss the intuition behind this result, relating it to time priority versus price priority. I.e. why would there be no effect of fragmentation if the tick size were nil?

Suggested solution. Fragmenting the market essentially reduces time priority, thus increasing the incentives to post new limit orders at any given tick (since they will execute with a higher probability). In a static analysis, this leads to more depth, whereas the dynamic effect may be to make it less attractive to post the 'early' limit orders, thus increasing the time it takes to rebuild liquidity after large market orders.

(c). Biais, Foucault and Moinas (2011) say that "(The equilibrium) P3 generates "crowding out" since slow institutions are sidelined and only fast institutions trade. Hence only a small fraction of the potential gains from trade can be reaped. Unfortunately, such equilibria can be pervasive." Explain intuitively how crowding out can be an equilibrium when there are potential gains from trade, and discuss whether this is realistic.

Suggested solution. Crowding out occurs through a self-fulfilling process where the market maker expects a great deal of adverse selection and therefore sets a large spread. The large spread then crowds out investors with moderate information, leading to the expected adverse selection. In reality, such equilibria seem less likely. A competing market maker may be able to make a profit by setting a price that attracts more informed traders (since they will buy at best price), so competition seems to favor the most inclusive equilibrium.

Problem 2

We consider an extension of the Glosten-Milgrom model that allows us to analyze speculation. In particular, we will allow for resale, but to maintain tractability we do so in a specific way.

To be concrete, suppose that we are in a Glosten-Milgrom setting where an asset has an unknown value: $v = -1$ or $v = 1$, with equal probability. There are three types of traders: informed traders (type I) know v and act as risk-neutral profit maximizers; uninformed rational traders (type U) do not know v and act as risk-neutral profit maximizers; noise traders (type N) buy and sell the asset with equal probability.

The game has two periods. To simplify matters, we assume that there are three cases:

- Case **A**: In the first period a type-I trader arrives and in the second a type-N trader. This case occurs with probability α.
- Case **B**: In the first period and second period the same type-U trader trades. This occurs with probability β.
- Case **C**: In the first period a type-N traders arrives and in the second period a different type-N trader arrives. This occurs with probability $1 - \alpha - \beta$.

Nature draws a case at the beginning of the game.[1] The case is known to the traders, for instance, in case **B** the trader knows that he is a type-U trader and that he will trade in both periods.

A risk-neutral market maker sets competitive (zero-profit) ask and bid prices in each period t, denoted a_t and b_t, respectively. In contrast to the traders, he does not observe which case has been drawn, and thus does not know the type of the traders he is facing. Finally, assume that in case **B**, the type-U traders do not own any units of the asset at the beginning of period 1, and they cannot go short – i.e. sell an asset they do not have.

We want to analyze whether in case **B** the uninformed trader would ever have an incentive to speculate, i.e. to buy in period 1 with a view to selling in period 2, in spite of his lack of information about the asset's value.

(a). Argue that in case **A**, a type-I trader with $v = 1$ will always buy in period 1 in equilibrium, whereas a trader who saw $v = -1$ will never buy.

Solution. Standard argument.

(b). We start with period 1. Let us find the ask price. Notice that the expected asset value conditional on case **B** or **C** is zero, whereas conditional on case **A** it is 1 (viz. discussion in (a)), so[2]

$$\begin{aligned} a_1 &= \mathbb{E}[v|buy_1] \\ &= \mathbb{P}(\mathbf{A}|buy_1) \cdot (1) + \mathbb{P}(\mathbf{B}, \mathbf{C}|buy_1) \cdot (0). \end{aligned}$$

Bayes' Rule yields

$$\mathbb{P}(\mathbf{A}|buy_1) = \frac{\alpha \mathbb{P}(buy_1|\mathbf{A})}{\alpha \mathbb{P}(buy_1|\mathbf{A}) + \beta \mathbb{P}(buy_1|\mathbf{B}) + (1 - \alpha - \beta)\mathbb{P}(buy_1|\mathbf{C})}.$$

Assume that U types always buy in case **B**: $\mathbb{P}(buy_1|\mathbf{B}) = 1$. Using the above results,

[1] The cases can be interpreted as follows: in case **A**, a rational informed trader arrives and trades based on his information. Since he will have no reason to trade again in the model, he exits the market place and any further trade is noise trade. In case **B**, an uninformed trader enters. If he trades, it must be for speculative reasons since he has no information. Therefore, he must trade again in order to make a profit. In case **C**, we have pure noise trade.

[2] Here, buy_1 denotes a buy in period 1.

show that

$$a_1 = \frac{\alpha}{1+\beta}.$$

Solution. Filling out the conditional probability we get

$$\frac{\alpha\frac{1}{2}}{\alpha\frac{1}{2} + \beta + (1-\alpha-\beta)\frac{1}{2}} = \frac{\alpha}{1+\beta}.$$

(c). Now we move to period 2. Suppose that in case **B** the U type always sells the asset: $\mathbb{P}(sell_2|\mathbf{B}) = 1$. Let us find b_2 conditional on a buy in period 1, and denote this $b_2|_{buy}$. We know from our previous analysis that it is impossible that a I type with $v = -1$ would have bought in period 1. So $b_2|_{buy} = \mathbb{P}(\mathbf{A}|buy_1, sell_2)\cdot(1) + \mathbb{P}(\mathbf{B}, \mathbf{C}|buy_1, sell_2)\cdot(0)$. Show that

$$b_2|_{buy} = \frac{\alpha}{1+3\beta}.$$

Solution. Now, since $\mathbb{P}(buy_1, sell_2|\mathbf{A}) = \mathbb{P}(sell_2|\mathbf{A}, buy_1)\mathbb{P}(buy_1|\mathbf{A}) = \frac{1}{2}\frac{1}{2}$, and similarly $\mathbb{P}(buy_1, sell_2|\mathbf{C}) = \frac{1}{2}\frac{1}{2}$, we get

$$\begin{aligned} b_2|_{buy} &= \mathbb{P}(\mathbf{A}|buy_1, sell_2) \\ &= \frac{\alpha\mathbb{P}(buy_1, sell_2|\mathbf{A})}{\alpha\mathbb{P}(buy_1, sell_2|\mathbf{A}) + \beta\mathbb{P}(buy_1, sell_2|\mathbf{B}) + (1-\alpha-\beta)\mathbb{P}(buy_1, sell_2|\mathbf{C})}. \end{aligned}$$

Filling out the conditional probability we get

$$\frac{\alpha\frac{1}{2}\frac{1}{2}}{\alpha\frac{1}{2}\frac{1}{2} + \beta + (1-\alpha-\beta)\frac{1}{2}\frac{1}{2}} = \frac{\alpha}{1+3\beta}.$$

(d). In case **B**, the U-type trader has no information about fundamentals, but he has private information in the sense that he knows his own type. Can he use this to make a profit? I.e. given a_1 and $b_2|_{buy}$, would he ever speculate in buying the asset in period 1 and selling it in period 2? Explain the intuition for this.

Solution. No, $a_1 < b_2|_{buy}$ implies that he would have negative expected (and realized) profits. Knowing his own type is worthless since he would need to first make the market believe that he is a noise trader, and then that he is an informed trader, in order to reap a profit. This he cannot do in this setup.

(e). Given your answer to the previous point, what will be the *equilibrium* behavior of the traders, and the *equilibrium* prices $(a_1, b_2|_{buy})$ of the model?

Solution. Since U-type traders will not trade, we will have $a_1 = \frac{\alpha/2}{\alpha/2+(1-\alpha-\beta)/2} = \frac{\alpha}{1-\beta}$ and $a_2 = \frac{\alpha/4}{\alpha/4+(1-\alpha-\beta)/4} = \frac{\alpha}{1-\beta}$. Similarly, $b_1 = b_2$. The second period reveals no new information - only noise trade will occur.

(f). Suppose now we introduce some noise into the model in the form of a public signal in period 2. Denote this signal by z. We make the following assumptions. In case **A**, the signal is fully informative: $z = v$. In case **B**, the signal is pure noise: $z = 1$ or $z = -1$ with equal probability and z is independent of v. In case **C**, there is no signal.[3]

Now a_1 and b_1 are the same as before. But period-2 prices will depend not just on the period-1 order, but also on the realization of the signal. Argue that $\mathbb{P}(\mathbf{C}|z = 1) = \mathbb{P}(\mathbf{C}|z = -1) = 0$. Suppose that in case **B** the type-U trader buys in period 1 and sells in period 2. Calculate the period-2 bid price conditional on a buy in period 1 and $z = 1$: $b_2|_{buy,z=1}$. Argue that $b_2|_{buy,z=-1} = 0$.

Solution. We must have $\mathbb{P}(\mathbf{C}|z = 1) = \mathbb{P}(\mathbf{C}|z = -1) = 0$ since a non-empty signal reveals that we are either in case **A** or **B**.

Since noise traders are revealed not to be present, and since $\mathbb{P}(z = 1|\mathbf{A}, buy_1, sell_2) = 1$ and $\mathbb{P}(z = 1|\mathbf{B}) = 1/2$

$$b_2|_{buy,z=1} == \frac{\alpha\frac{1}{2}\frac{1}{2}(1)}{\alpha\frac{1}{2}\frac{1}{2}(1) + \beta(\frac{1}{2})} = \frac{\alpha}{\alpha + 2\beta}.$$

Finally, $b_2|_{buy,z=-1} = 0$ because in case **A**, an I type would never buy in the first period if $z = -1$, leaving case **B** as the only possible case. The expected value conditional on a U type is zero.

(g). Finally, show that for certain parameter values α and β, it will be profitable for U types to speculate in case **B**: i.e. to buy in period 1 and sell in period 2. What is it that

[3]You may think of this as the signal taking some third value, e.g. $z = \emptyset$.

permits them to speculate now?

(Hint. You must calculate the expected bid price in period 2.)

Solution. The expected profit from speculating for a U type is

$$\frac{1}{2}(b_2|_{buy,z=1} - a_1) + \frac{1}{2}(b_2|_{buy,z=-1} - a_1) = \frac{1}{2}b_2|_{buy,z=1} - a_1 = \frac{\alpha(1-2\alpha-3\beta)}{2(1+\beta)(\alpha+2\beta)}$$

This is positive if $2\alpha + 3\beta < 1$.

Now, the U type has extra information, in the sense that he knows that the signal is noise. Hence, he can speculate in the signal confirming his buy order, leading the market maker to update his belief that he is an informed trader. In the case of a bad signal, his downside is limited since it will just be revealed that he is uninformed.

Short-selling matters in the sense that without the constraint, it would be optimal for him just to wait until period 2 and then go short if the signal is positive.

Problem 3

Below is an article from the Financial Times on February 15, 2015. Please write a short essay discussing to which extent the course readings can relate to the issue of this text. In particular, consider the theories exposed in lecture 13, but feel free to include theories from other parts of the course (for instance the part on liquidity and asset prices). You are also welcome to elaborate your answer beyond the syllabus.

"A decade ago, the market for EM (emerging markets) hard currency corporate bonds hardly existed. Today, it is bigger than the US high-yield corporate bond market, an asset class familiar to investors for decades, and more than four times the size of Europe's high-yield bond market.

What has driven such extraordinary growth? In just a few years before the global financial crisis of 2008-09, emerging markets won over the world's investors. In 2001, Goldman Sachs identified the Bric economies – Brazil, Russia, India and China – as the new engines of global growth. Chinese demand drove a commodity boom that helped billions of people rise out of poverty and into the consuming classes.

But with Brazil's economy imploding, China slowing and dark shadows over markets from Venezuela to Russia and Ukraine, some analysts worry that the party has gone on too long.

Stuart Oakley, global head of EM foreign exchange trading at Nomura in London, points to how easily things could go wrong. "It is entirely possible that we could see a default by a big, emerging market commodity exporting corporate," he says.

"In that scenario you would get people redeeming money from big EM asset managers, bids for the bonds from banks would dry up, there would be sharp price drops on those and all associated assets and a sell-off across this or another asset class."

Fears of a rout

One source of danger is that the EM corporate bond market, pumped up by years of often indiscriminate buying, is still being engorged by a search for yield among global investors. It is also showing alarming signs of distress at a time when the ability of the financial system to handle trades between buyers and sellers is much reduced, increasing the risk that any sudden exit could quickly turn into a disorderly rout.

(...)

As noted recently by Zoltan Pozsar, a former senior adviser at the US Treasury, while the yield on the benchmark US Treasury bond has fallen from 6 per cent in 2000 to less than 2 per cent today, the returns sought by many US public pension funds have barely changed at about 8 per cent. Other big institutional investors also have imperatives that are hard to satisfy by investing in what are usually seen as safe assets.

The result is known as "forced buying" – asset managers buying assets outside their usual area of expertise because they have to put their clients' cash to work somewhere.

(...)

This has led to a process that Sergio Trigo Paz, head of EM debt at BlackRock, calls shut your eyes and buy. Many crossover investors, who are new and often far from committed to emerging markets, have driven up the price of some bonds even as risks have become more apparent.

(...)

Flight to quality

The greater danger is that investors start to leave the asset class altogether. That could be triggered by a default, but also by a much lesser event. If a bond falls sharply in price, any investor who has borrowed money to buy it – as hedge funds habitually do – will have to sell others to make up the loss. Such waves of selling can spread quickly, not only to other bonds but also to other asset classes.

(...)

Quantitative easing has pumped up the primary markets but, since the financial crisis, regulatory and other changes have caused a drought of liquidity on secondary markets. Investment banks that used to hold large inventories of bonds on their books can no longer do so. Analysts at UBS say the volume of assets held by banks is half the level of five years ago, while the volume of assets held by investors is four times what it was.

“When there are bouts of buying there are no sellers and when there are bouts of selling there are no buyers,” says Mr Spegel. “It creates the perfect environment for distressed markets to get worse. This is the year of negative feedback loops.”

(...)

“Last year,” he says, “everything that could go wrong, did go wrong. China slowed down, commodity prices fell, we had QE (quantitative easing) tapering, the Ukraine crisis, Brazil blowing up – and the return on EM corporate bonds was 5 per cent. It is a very well diversified market.”

Nevertheless, others worry that the growth of the EM corporate asset class is a clear example of a bubble, one that is being blown up by the apparently unending tide of QE.

“The point of QE is to inflate the real economy,” says Mr Oakley at Nomura. “But instead of driving growth it is creating asset bubbles. The danger is that it will drive bubbles until they burst.” (...)”

Suggested solution. There are several issues to discuss in this article.

- The first theme of lecture 13 was herding. The comments on ‘indiscriminate buying’ sound like herding behavior. It is conceivable that that search for yields set of a chain-reaction in investment. However, this does not seem so much like an information-driven cascade, but more an agency-driven cascade, as suggested by the comment on ‘forced buying’. Asset managers are making investments in areas where they have little expertise in order to satisfy client demands.

- The second theme of lecture 13 was speculative bubbles and the role of second-order beliefs and the role of seemingly small events in coordinating the actions of the market.

There seems to be a common beliefs that the market looks a great deal like a bubble. Many investors may be convinced that this is the case, but choosing to wait with selling out for speculative reasons. Events that by themselves are not catastrophic, such as isolated defaults, may serve to coordinate the actions of these investors and cause a crash.

- Another concern is that secondary market liquidity is changing for the worse, implying that these investments carry a significant liquidity risk. Referring to the theories exposed in lecture 9 on liquidity premia, this should exert a downward pressure on prices, something which may cause a downward spiral as more investors leave the market and liquidity dries up even more.

- There is also the link between the primary market and the secondary market. The primary market has been inflated by quantitative easing, and while we have not seen a specific theory that says that high primary market prices should lead to high secondary market prices, it seems conceivable that there is a link. In that case, putting and end to quantitative easing also means deflating secondary markets, which could interact with the above effects.